The general behavior of NLO unintegrated parton distributions based on the single-scale evolution and the angular ordering constraint

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Abstract

To overcome the complexity of generalized two hard scale (k_t,μ) evolution equation, well known as the $Ciafaloni$, $Catani$, $Fiorani$ and $Marchsini$ ($CCFM$) evolution equations, and calculate the unintegrated parton distribution functions ($UPDF$), $Kimber$, $Martin$ and $Ryskin$ (KMR) proposed a procedure based on (i) the inclusion of single-scale (μ) only at the last step of evolution and (ii) the angular ordering constraint (AOC) on the $DGLAP$ terms (the $DGLAP$ collinear approximation), to bring the second scale, k_t into the $UPDF$ evolution equations. In this work we intend to use the $MSTW2008$ (Martin et al) parton distribution functions (PDF) and try to calculate $UPDF$ for various values of x (the longitudinal fraction of parton momentum), μ (the probe scale) and k_t (the parton transverse momentum) to see the general behavior of three dimensional $UPDF$ at the NLO level up to the LHC working energy scales (μ^2). It is shown that there exits some pronounced peaks for the three dimensional $UPDF$ ($f_a(x, k_t)$) with respect to the two variables x and k_t at various energies (μ). These peaks get larger and move to larger values of k_t, as the energy (μ) is increased. We hope these peaks could be detected in the LHC experiments at $CERN$ and other laboratories in the less exclusive processes.

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I. INTRODUCTION

To understand the event structure observed in different laboratories i.e. $SLAC$, $HERA$, $DESY$ etc, and especially the one would be expected in LHC ($CERN$), the theoretical formalisms which describe the small x (x is $Bejorken$ variable) region are vital. The main unknown parameters in these models are the unintegrated parton distribution functions ($UPDF$) [1–4]. The $UPDF$ are two-scales dependent distributions which are functions of x (longitudinal momentum fraction of the parent hadron) and the scales k_t^2 and μ^2, the squared transverse momentum of the parton and the factorization scale, respectively. As we pointed out these distributions are the essential ingredients for the less exclusive phenomenological computations in the high energy collisions of particle physics.

It is well known that in the region of high energy and moderate momentum transfer i.e. small x, the collinear factorization theorem i.e. *Dokshitzer-Gribov-Lipatov-Altarelli-Parisi* ($DGLAP$) [5–8] evolution, breaks down. This happens because of the large increase of the phase space available for the gluon emissions (i.e. a rapid rise in the gluon density), which makes the quantum chromodynamics (QCD) perturbative expansions unjustified and one can not obtain the $UPDF$. On the other hand, at above high energy limit, the cross section can be predicted by using the k_t factorization and the *Balitsky-Fadin-Kuraev-Liptov* ($BFKL$) [9–11] evolution. But the precision of k_t factorization is not good e.g. the next-to-leading order (NLO) corrections to $BFKL$ are very large [12–15]. Another approach to derive the $UPDF$ is the *Ciafaloni-Catani-Fiorani-Marchesini* ($CCFM$) equations [16–20]. Although the $CCFM$ equations describe the evolution of the $UPDF$ correctly, but working in this framework is a complicated task, so practically they are used only in the Monte Carlo event generators [21–25]. On the other hand, up to now, there is not a complete quark version for these kind of equations [16–20, 26], since the enhanced terms that are resumed by $CCFM$ come from gluon evolution. However, to over come this problem, it has been shown that the $CCFM$ equation can be reformulated (the linked dipole chain model) by reducing the division between the initial and the final state radiation diagrams using the colour dipole cascade model [27–29].

The *Kimber*, *Martin* and *Ryskin* (KMR) [30] approach is an alternative prescription

for producing the $UPDF$ which is based on the standard $DGLAP$ equations [5–8],

$$\frac{\partial a(x,\mu^2)}{\partial \ln(\mu^2)} = \sum_{a'=q,g} P_{aa'} \otimes a'(y,\mu^2), \tag{1}$$

where $a(x,\mu^2) = xq(x,\mu^2)$ or $xg(x,\mu^2)$ and $P_{aa'}(z)$ are the conventional (integrated) parton distribution functions (PDF) and the well known $DGLAP$ splitting functions, respectively. In equation (1) the symbol $\otimes$ denotes a convolution as,

$$f \otimes g = \int_x^1 \frac{dy}{y} f(\frac{x}{y}) g(y). \tag{2}$$

In this approach under the certain approximation the $UPDF$ are obtained from the PDF by introducing the scale μ only in the last step of evolution with the inclusion of angular ordering constraint (AOC). It has been shown that the KMR prescription gives the same results both for the $DGLAP$ and the unified $BFKL$-$DGLAP$ equations [31] and the AOC is applicable to all orders as in the $CCFM$ formalism, i.e. all the loops contributions via the chain of evolution which are restricted by AOC, are resumed.

In this work, along the lines of our recent calculations [33, 34], we intend to use the KMR prescription with $MSTW2008$ [32] PDF to produce three dimensional plots of $UPDF$ at different energies (μ) and discussed the various behavior of $UPDF$ i.e. $f(x,k_t,\mu)$. So the paper is organized as follows: In section II we briefly introduce the KMR formalism and finally, section III is devoted to the results and the discussions concerning the three dimensional (3D) graphs of the $UPDF$ produced via this approach.

II. THE KMR FORMALISM [30]

The KMR prescription [30] works as a machine that by taking a defined PDF as inputs, generates $UPDF$, as outputs. Using the leading order (LO) splitting functions, $P_{aa'}$, the $DGLAP$ equations can be written in a modified form as [30],

$$\frac{\partial a(x,\mu^2)}{\partial \ln(\mu^2)} = \frac{\alpha_s}{2\pi}\left[\int_x^{1-\Delta} P_{aa'}(z)\, a'\left(\frac{x}{z},\mu^2\right) dz - a(x,\mu^2)\sum_{a'}\int_0^{1-\Delta} dz' P_{a'a}(z')\right], \tag{3}$$

where Δ is a cutoff to prevent $z=1$ singularities in the splitting functions arising from the soft gluon emission. In the conventional $DGLAP$ formalism, $\Delta = 0$ and the singularities are canceled by the virtual terms. The value of Δ can be determined by imposing an appropriate

dynamical condition which is replaced by the angular ordering constraint arising from the coherency of the gluon emissions [35, 36],

$$... > \theta_n > \theta_{n-1} > \theta_{n-2} > ..., \tag{4}$$

where θ's are the radiation angels. This condition, at the final step of evolution, leads to [16–19, 31],

$$\mu > \frac{zk_t}{1-z} \Rightarrow \Delta = 1 - z_{max} = \frac{k_t}{\mu + k_t}. \tag{5}$$

The first part of the equation (3), shows the contribution of real emissions, that can change the transverse momentum k_t. The second term expresses the evolutions due to the virtual effects without changing the k_t. The latter can be re-summed, to obtain a survival probability factor,

$$T_a(k_t, \mu) = \exp\left[-\int_{k_t^2}^{\mu^2} \frac{\alpha_s({k'_t}^2)}{2\pi}\frac{d{k'_t}^2}{{k'_t}^2}\sum_{a'}\int_0^{1-\Delta} dz' P_{a'a}(z')\right]. \tag{6}$$

Now, similar to the *Sudakov* form factor, the above survival probability, equation (6), is imposed into the equation (1), and by using equation (2), we find the equation which describes the $UPDF$,

$$\begin{aligned} f_a(x, k_t^2, \mu^2) \; &= \; T_a(k_t, \mu)\left[\left.\frac{\partial\, a(x, \mu^2)}{\partial\, ln(\mu^2)}\right|_{\mu^2 = k_t^2}\right]_{real} \\ &= \; T_a(k_t, \mu)\frac{\alpha_s({k_t}^2)}{2\pi}\int_x^{1-\Delta} P_{aa'}(z)\, a'\left(\frac{x}{z}, {k_t}^2\right) dz. \end{aligned} \tag{7}$$

More explicit forms of the above equation for the gluon g and the different quark flavors $q = u, d, s, ...$ are as follows,

$$\begin{aligned} f_q(x, k_t^2, \mu^2) \; &= \; T_q(k_t, \mu)\frac{\alpha_s({k_t}^2)}{2\pi} \\ &\times \int_x^{1-\Delta} dz \left[P_{qq}(z)\frac{x}{z}\, q\left(\frac{x}{z}, {k_t}^2\right) + P_{qg}(z)\frac{x}{z}\, g\left(\frac{x}{z}, {k_t}^2\right)\right], \end{aligned} \tag{8}$$

and

$$\begin{aligned} f_g(x, k_t^2, \mu^2) \; &= \; T_g(k_t, \mu)\frac{\alpha_s({k_t}^2)}{2\pi} \\ &\times \int_x^{1-\Delta} dz \left[\sum_q P_{gq}(z)\frac{x}{z}\, q\left(\frac{x}{z}, {k_t}^2\right) + P_{gg}(z)\frac{x}{z}\, g\left(\frac{x}{z}, {k_t}^2\right)\right]. \end{aligned} \tag{9}$$

The key observation here is the dependency on the scale μ^2, which appears at the last step of the evolution. Another point is that, the *Sudakov* form factor which arises from the resumption of virtual effects, can be used at every order of approximation. Although the splitting functions must be used at the NLO level, but as it is shown in [37], the NLO corrections to the splitting functions, are relatively small in comparison to the LO contributions. However, as stated above, only the LO splitting functions are used. On the other hand, although the definition of *Sudakov* form factor (like the PDF themselves) has been started intuitively from a probabilistic interpretation, but its role in the mathematical description of the evolution remains in the equations.

The primary computations based on this kind of approach to evaluate the $UPDF$, show very good agreement with the experimental data for F_2 [30]. Also, in recent years, the KMR prescription have been widely used for phenomenological calculations (see [33] and the references therein). Recently the stability and the reliability of the KMR $UPDF$ have been investigated in [33, 34].

Finally, we should mention here that, the key property of the $CCFM$ approach (as given in their publications [16–22]) is the AOC, which in turn has root in the coherency of gluon radiation along the evolution chain, that is valid for whole range of x values. In the conventional $DGLAP$ formalism, the strong ordering constraint on the transverse momenta, restricts the domain of study to the large and moderate values of x:

$$\hat{\sigma}(\gamma^* q \to qg) = \int_{p_{t_{min}}}^{p_{t_{max}}} dp_t^2 \frac{d\hat{\sigma}}{dp_t^2},$$

where

$$p_{t_{min}}^2 = \lambda^2,$$

and

$$p_{t_{max}}^2 = p_t^2|_{\sin^2\theta=1} = k'^2 = \frac{\hat{s}}{4} = Q^2 \frac{1-x}{4x}.$$

So to obtain the $DGLAP$ equations with $\ln(\frac{\hat{s}}{4}) \simeq \ln(Q^2)$, x should not be very low. In the KMR prescription the AOC property of the $CCFM$ formalism is applied to modified $DGLAP$ evolution as a cut off on the integrals. Therefore, the results of these modifications show that the effect of application of AOC is even more important than the inclusion of the conventional low x effects in the $BFKL$ approach [30].

III. RESULTS AND DISCUSSION

As we stated in the section II, by using the equations (8) and (9), the $UPDF$ are generated via the KMR procedure. For the input PDF, the $MSTW2008$ [32] set of partons at the NLO level are used [40]. Since the generated $UPDF$ ($f_a(x, k_t^2, \mu^2)$) are three variable functions, by fixing the scale μ^2, their values versus x and k_t^2 are plotted in the various panels of figures 1, 2, 3 and 4 for the gluons, the up, the strange and the bottom quarks, respectively. For the better comparison, the values of the μ^2 are chosen in a wide range $\mu^2 = 10, 10^2, 10^4, 10^8\ GeV^2$ which is up to the LHC working scales. The three typical quark flavors, the u quarks consists of the valence and the sea contributions $u = u_v + u_{sea}$ and the s and the b quarks which are completely sea distributions, are presented. The main feature of these figures is exhibiting the general behavior of the $UPDF$ with respect to the coupled contributions of x and k_t^2. For example, the most probable value of k_t^2 (x) at every x (k_t^2) for any kind of partons can be checked. As it can be seen, by increasing the scale μ^2 the graphs are shifted to the higher k_t^2. This is expected, since the probability of finding partons with larger k_t^2 is more probable at higher scales. The growth of the values of the distributions by increasing μ^2 and decreasing x and also the phenomenon of converging the quark distributions to a unique value at small x are known characteristics of the parton distributions which are the heritage of their parent PDF. The different behaviors of up and strange quarks at large x have root in the valence contribution in the case of up quark. The pronounced peaks become wider with respect to k_t^2, and move to higher values of k_t^2. This behavior is much effective for the up, the strange and the bottom quarks. The peaks come from the concept of distributions and they are results of the dynamical evolution of partons. The figures show that at given values of hard scale and x, at which k_t, it is more probable to detect the out going partons. So based on the final partons, we can predict the dynamical properties of the produced jets and their components, and on the other hand it can inform us about the precision of the current theoretical formalisms itself. The input PDF of $MSTW2008$ are also given in the figure 5, for comparison. With good approximation by integrating over $UPDF$, we can get the input $MSTW2008$ PDF ($a(x, \mu^2) = \int^{\mu^2} \frac{dk_t^2}{k_t^2} f_a(x, k_t^2, \mu^2)$). For example for gluons, at $x = 0.01$ and $\mu^2 = 100$ we get 6.7 whereas $MSTW2008$ gives the value of 6.5 i.e. 3% off. Situations are the same for other points and parton distributions. It is worth to say that in the original KMR work, they get

25% discrepancies [30] for above comparison. This is also evident by comparison of figure 5 with those of 1 to 4 i.e. the $UPDF$ are decreasing by increasing x. On the other hand, as have been discussed in the KMR and other related works, because of the imposition of angular ordering, the $UPDF$ have values for $k_t^2 \geq \mu^2$ as x decreases. But this will not affect the above integration too much. The figures 1 to 4 also show that, for low scales ($\mu^2 \simeq 10\ GeV^2$) the $UPDF$ become negative when x becomes close to one. This reflects the negative values of $MSTW2008$ gluon distributions at the NLO level and beyond that. So the negative values of $UPDF$ have root in the parent integrated gluon distributions which in turn are the result of $MSTW2008$ assumptions [32]. As it was pointed, in the $MSTW2008$ [32], for better data fitting it is allowed that, the gluon distribution takes negative values, because there is no theorem that imposes positivity condition on PDF beyond the LO approximation. So they become negative in order to fit the data (in other words they can be traced to the slow evolution of F_2 at small x and Q^2 i.e. a positive gluon would give too rapid evolution to fit the $dF_2/d\ln(Q^2)$ data. Then in the KMR integrals, the evaluation of input $g(x, k_t^2)$ at small x and k_t (as a scale, instead of Q^2 in $g(x, Q^2)$) leads to the negative values for the output $UPDF$. Finally, (i) the comparison of $UPDF$ produced from different PDF sets have been made in our former works [33, 34] . The different parameterizations procedures lead to different PDF, and a discussion about these procedures is presented in [33, 34] and references therein. (ii) The differences between the LO and the $NLO\ PDF$ are parameterizations dependent. In the $MSTW2008$ this is noticeable, but in some other parameterizations sets based on different assumptions and procedures it can be less (e.g GRV sets [33, 34]), but as we have showed in [33, 34] (by investigating the ratios of KMR $UPDF$ compared to the corresponding ratios of input PDF) the relative differences are less in the output $UPDF$ and the KMR prescription suppresses these discrepancies. To show this point more transparently, in figure 6 we have plotted the gluon $UPDF$ with three different input PDF, namely the original KMR [30] with $MRST99$ [38] PDF, our recent works [33, 34] with $GJR08$ PDF [39] and present calculation ($MRST2008$) at $\mu^2 = 100$ GeV^2 and $x = 0.1, 0.01, 0.001$ and 0.0001 in terms of k_t^2. It is clearly seen that different input PDF give very similar $UPDF$. (iii) In fact a complete prescription for producing the $NLO\ UPDF$ needs to include both the PDF and the splitting functions at the NLO level. This prescription is presented in [37], but as it is shown in this reference [37], inclusion of the NLO splitting functions have very low effect comparing to the contribution of the NLO

PDF. Therefore, ignoring the corrections due to the NLO splitting functions do not affect our analysis of the general behavior of the NLO $UPDF$. (iv) There is no restriction on the k_t dependency. As the orders of the approximation are in terms of orders of $\alpha_s(k_t^2)$, the NLO accuracy is contained in the NLO PDF and splitting functions that discussed in the former comments. Hence at scales $k_t^2 \geq Q_0^2$,where Q_0^2 is the scale that upper than it, the perturbative QCD is still applicable, these results are valid.

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FIG. 1: The unintegrated gluon distribution functions generated by the KMR procedure with the fixed values of $\mu^2 = 10, 10^2, 10^4, 10^8\ GeV^2$.

FIG. 2: As figure 1 but for the up quark.

FIG. 3: As figure 1 but for the strange quark

FIG. 4: As figure 1 but for the bottom quark

FIG. 5: The NLO integrated parton distribution function of $MSTW2008$ versus x for the fixed values of $\mu^2 = 10, 10^2, 10^4, 10^8$ GeV^2

FIG. 6: The $UPDF$ of $MSTW2008$ (present calculation, dotted curve), $MRST99$ (dash curve) and $GJR08$ (full curve). See the text for details.